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DYNAMICAL BRANCHING

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ABSTRACT
We investigate the Branching space-times logic in a dynamic setting. We present a new logic, called Dynamic branching logic and show some of its basic properties.
Keywords: branching time, branching space-times, dynamic logic

1. Introduction

Branching temporal logics, those originating from Branching space-times (BST), are attempting to model time. Although branching should capture the changes of possibilities, the point-events that constitute these branches are given in advance and in a tenseless manner. Common view of time and changes, however, is more connected to dynamic evolution of options, possibilities, and the future. For this reason, we find it suitable to try a synthesis of dynamic and branching logic.[1] We will shortly discuss the philosophical background of our work, present the original BST approach, and then present a dynamical branching time model.

2. Philosophical background

How does our work approach the questions of eternalism or presentism, determinism or indeterminism?

Let us define the terms clearly:

> *Eternalism* is committed to the tenseless coexistence of all events, while *presentism* is committed to the thesis that existence is confined just to the present events, while future and past events do not exist.
>
> (Dorato, 2012)

From these definition stems also our motivation. We hold the view in this paper that a present event can give rise to the following event by a dynamic process and this would be a presentist model, as opposed to the static eternalist model.

[1] The idea to approach branching as a dynamic process was suggested to the author by Ondrej Majer at a session of the 'Prague dynamic group' (O. Majer, M. Peliš and others).

The idea of determinism is more complex than it might seem at first sight and we can differentiate between approaches based on what exactly we mean by determinism (Müller & Placek, 2015). Nevertheless, we use the following definition of determinism: "given the state of the world now, there is only one possible future outcome". Thus, indeterminism means that given the present state we have multiple possible future outcomes.

Our theory would like to be a presentist indeterministic theory. Therefore it would allow multiple possible outcomes from present events without committing to the existence of any future or past events. Now, as we do not wish to ignore (entirely) advances made in the field of physics, we need to address the question of 'present'. The arrival of Special Theory of Relativity (STR) marked also the end of a simple idea of present or simultaneity[2] (Dieks, 1988). It showed how closely time and space are interwoven. We do not want to ignore this and therefore get inspiration from the Machian view of Barbour (Barbour, 2000) that is consistent with STR. While BST uses the notion of space-time fully, we will use a space and time approach. This dichotomy could be viewed as a version of the endurantism and perdurantism discussion[3] or it can be considered simply as a different formulations of eternalism and presentism (Dorato, 2012). Instead of taking entities as entities with four dimensions, three spatial and one temporal, as seen in STR or BST, we consider entities to be only three dimensional. Therefore our basic building blocks are not space-time events, but 3D configurations of 3D entities. We could, in order to accommodate the results of STR, discuss the role of observers in these configurations. However, at this point, we focus merely on the introduction of some ideas how such presentist approach could start out and do not venture deeper into this topic. An observer-related[4] dynamics was discussed already for BT. The so called BT+I+AC[5] structures are introduced in (Belnap, Perloff, & Xu, 2001). Nevertheless, these structures focus on the establishment of an agents context and not on dynamic changes and therefore they stay out of our focus in this paper. Mentioning another related work, Müller introduced transitions, in the basic case transitions from a point-event to its immediate outcomes, as a framework to investigate causation and probability theory. The approach uses consistency as a way how to define causal possibility. Still, transitions are again built on the usual BST foundations and transitions are more like tool choosing selected point-events rather than dynamically constructing them.

3. Branching space-times

BST is important for us to keep in mind as a reference theory whose expressiveness we would like to match but do so in a different way. Let us have a look at how time and possibilities are treated there.

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2 The notion that two spatially separated events happen at the same time.

3 Endurantism expresses the view that objects are 3D material objects wholly present at every moment of their existence. Perdurantism is the somewhat opposite view that objects are 4D objects whose existence extends over a period of time, i.e. the objects have different temporal parts.

4 Precisely an agent-related.

5 Branching time + instants + agents and choices.

Branching space-times were introduced by Belnap in (Belnap, 1992). However, we use the concise formulation from Placek and Wroński article (Placek & Wroński, 2009). The basic definitions connected to BST follow.

Definition 1 (Placek & Wroński, 2009)

The set OW called Our World , is composed of point-events e ordered by $\leq$.

A set $h \subseteq OW$ is upward-directed iff $\forall e_1, e_2 \in h \, \exists e \in h$ such that $e_1 \leq e$ and $e_2 \leq e$.

A set h is maximal with respect to the above property iff $\forall g \in OW$ such that $h \subset g$, g is not upward-directed.

A subset h of OW is a history iff it is a maximal upward-directed set.

For histories h_1 and h_2, any maximal element in $h_1 \cap h_2$ is called a choice point for h_1 and h_2.

Histories are meant to capture the familiar notion of possible courses of events. Hence if some event e occurred in one course of events, it is inconsistent with a different event e', its counterpart from a different course of events. We see a history can be a very large set as it would contain a series of point-events from the 'origin' of time until its 'end'. The quotation marks remind us that a model of BST can be without an origin and an end too.

Definition 2 (Placek & Wroński, 2009)

$\langle OW, \leq \rangle$ where OW is a nonempty set and $\leq$ is a partial ordering on OW is a structure of BST iff it meets the following requirements:

(1) The ordering $\leq$ is dense.
(2) $\leq$ has no maximal elements.
(3) Every lower bounded chain in OW has an infimum in OW.
(4) Every upper bounded chain in OW has a supremum in every history that contains it.
(5) (Prior choice principle) For any lower bounded chain $O \subset h_1 - h_2$ there exists a point $e \in OW$ such that e is maximal in $h_1 \cap h_2$ and $\forall e' \in O(e < e')$.

We can see the steps necessary to befriend branching and space-times. The notion of histories and their treatment is reminiscent of earlier branching temporal logic, namely computational tree logic and its fullpaths (Hodkinson & Reynolds, 2006). We see this approach as an eternalist one, assuming we have all point-events in OW and these eternally coexist. A different, not necessarily eternalist reading of BST was presented in (Pooley, 2013), where a non-standard non-eternalist interpretation of BST is presented as the "most promising way to reconcile becoming with relativity".

The language of Dynamic branching logic will use Priorean operators F, P (The meanings are as usual. In the order of the operators 'it will be true', 'it was true'.) and therefore we should also address the question, how are these interpreted in BST. We add one new operator, the '$Sett$:' operator denotes a settled option, i.e. true for all the branches. The $Sett$: operator allows us to basically describe the lack of options - in all courses of events, the statement will be true (for example if I want to capture the statement "no matter what I do, I will have some typos in this paper").

Definition 3 Point satisfies formula – BST (Placek & Wroński, 2009)

For the model $\mathfrak{M} = \langle OW, \leq, v \rangle$. Where v is the valuation v : Atoms $\rightarrow \mathcal{P}(OW)$. For a given event e and history h, such that $e \in h$:

$$
\begin{array}{llll}
\mathfrak{M}, e, h & \Vdash & p & \text{iff } e \in v(p) \\
\mathfrak{M}, e, h & \Vdash & \neg\varphi & \text{iff not } \mathfrak{M}, e, h \Vdash \varphi \\
\mathfrak{M}, e, h & \Vdash & \varphi \wedge \psi & \text{iff } \mathfrak{M}, e, h \Vdash \varphi \text{ and } \mathfrak{M}, e, h \Vdash \psi \\
\mathfrak{M}, e, h & \Vdash & F\varphi & \text{iff there is } e' \in OW \text{ and } e^* \in h \text{ s.t.} \\
 & & & \quad e' \leq e^* \text{ and } \mathfrak{M}, e', h \Vdash \varphi \\
\mathfrak{M}, e, h & \Vdash & P\varphi & \text{iff there is an } e' \in h \text{ s.t. } e' \leq e \text{ and } \mathfrak{M}, e', h \Vdash \varphi \\
\mathfrak{M}, e, h & \Vdash & Sett : \varphi & \text{iff for all } e' \in h', \text{ for all } h' \text{ such that } e \in h': \\
 & & & \quad M, e', h' \Vdash \varphi
\end{array}
$$

A short note about the future operator, although the expression does not contain the event e, it does contain the event $e*$ from the history h to which also e belongs. Therefore one could rephrase the expression as 'at a future event to e, namely $e*$, we will be able to say that φ is true'. We cannot, however, just pick event e' as it does not have to be necessarily in h and it might be that φ does not hold at $e*$ (i.e. it is not a settled future).

4. Dynamic branching logic

Dynamic branching logic (DBL) will attempt to model a presentist theory with indeterminism. Based on the definitions mentioned earlier, DBL is presentist in the sense that only a set of events (i.e. present events) is considered and possible future (or past) is constructed based only on these present events. Concerning indeterminism, DBL allows for present events to have multiple possible future outcomes (e.g. quantum observations). We present a syntactic and a semantic approach.

4.1 Syntax

Language of DBL is the following atomic statements ($a, b, c, ...$), truth and falsity ($\top$, $\bot$), logical connectives ($\wedge$), negation ($\neg$), temporal operators (F, P, G, H), actions ([C], [R], $[\varphi!]$), and the branching modality $Sett$:.

The atomic statements are assumed to be in present tense and simple, for example: 'The doctor is a time-traveling alien.' The temporal operators can be read as it is usual, i.e. $F\varphi$ stands for 'it will be true in the future that φ' and $P\varphi$ stands for 'it was true in the past that φ'. The temporal operators G and H represent then necessary temporal statements, i.e. $G\varphi$ means 'it will always be true that φ'.

DBL has three types of actions, [C] is construction, i.e. creation of the future, [R] is reconstruction, i.e. recreation of the past, and $[\varphi!]$ is claiming that a statement φ is true or becoming true (similar to public announcement from dynamic epistemic logics). The creation of the future and recreation of the past actions are inspired by Barbour's time capsules. The main idea can be summed up in the following way. Every state $s \in S$ contains evidence for the events that were in the past of the state (e.g. fossils, broken glass, contrail). This evidence allows us to reconstruct the events that lead to the given state and determine what possible states preceded the given state s. In a similar manner, the current state of affairs s determines (by physical laws) all the possible states. Therefore, based merely on our current state n, we can make a judgment about our past and future. The actions [C] and [R] then represent an ontological construction, not an epistemic one, from the given state. Because we do not have at this time a more detailed account of the states, the actions can be taken as functions from a state to states.

Because DBL aims to be also a model of branching, the idea of a statement being settled, i.e. true for all branches, is a useful one. We denote 'φ is settled' as $Sett : \varphi$.

The relation between the temporal operators is the following:

$$H\varphi \equiv \neg P \neg \varphi, G\varphi \equiv \neg F \neg \varphi \tag{1}$$

We can then look for axioms of our logic in well known modal logics and their temporal versions. As basis, we can take the minimal tense logic K_t (Hodkinson & Reynolds, 2006) (Goranko & Galton, 2015).

Axiom 4

The axioms for BTL are:

(1) all propositional tautologies
(2) $G(\varphi \to \psi) \to (G\varphi \to G\psi)$ and $H(\varphi \to \psi) \to (H\varphi \to H\psi)$
(3) $G\varphi \to GG\varphi$ and $H\varphi \to HH\varphi$
(4) $\varphi \to GP\varphi$
(5) $\varphi \to HF\varphi$
(6) $[\alpha](\varphi \to \psi) \to ([\alpha]\varphi \to [\alpha]\psi)$

Where α can be any action.

And the rules:

(1) Modus Ponens $\varphi, \varphi \to \psi \vdash \psi$
(2) $\varphi / G\varphi$
(3) $\varphi / H\varphi$
(4) $\varphi / [\alpha]\varphi$
(5) Substitution in any propositional tautology.

4.2 Semantics

We present a suggestion for DBL semantics. We need to first realize a DBL model represents a configuration of three dimensional objects, entities existing concurrently. Actions then allow us to construct a new model of a new instant discarding the old one. The model always represents the current status of temporal relations. Every model contains in itself also information about the future or past events. For example a model representing a configuration of the first of January 2015 will contain in itself also books describing past events, diaries of people, video tapes, or geological sediments. The configuration does also contain many predispositions for future events, for example an emitted electromagnetic signal or a sent letter. This approach is reminiscent of Barbour's time capsules (Barbour, 2000). We look at time as a static series of 3D images containing no time by their own virtue, but they do contain a lot of evidence connected to processes connected to time. Remember, however, neither past nor future exist. Merely the hints, predispositions, and conditions for the future are actualized.

The usual terminology speaks about states and this term does not part too much from our idea of a configuration. The main change is that we do not have point-events as our basic building blocks but states (configurations) of 3D objects. Therefore a state s truly represents all that is in the chosen 'snapshot' of the universe. It is not merely one point-event.

Definition 5
The relation $\prec$ is defined on S as the *causal ordering* of states.

We want to maintain the idea of branching similar to the one from BST and therefore we introduce also histories. However, these histories can be considered closer to the original histories from Branching time.
The relation $\prec$ is basically the usual accessibility relation. However, in our temporal context it is interpreted as the causal relation between states.

Definition 6
A frame $\mathcal{F} = \langle S, \prec \rangle$ is the *frame for DBL* iff
(1) $\prec$ is transitive
(2) there exists a state $n \in S$ such that it holds for all s from S that $s = n$ or there are $s_1, ..., s_k \in S$ such that $s_1 = s$ and $s_k = n$ and either $s_1 \prec s_2 \prec ... \prec s_k$ or $s_k \prec s_{k-1} \prec ... \prec s_1$
(3) for all $s_{past} \in S$ such that $s_{past} \prec n$ and for all $s_{future} \in S$ such that $n \prec s_{future}$ it holds that $s_{past} \prec s_{future}$

Hence the state n represents a connection point between the past and the future.

Definition 7
A (dynamic) *history* is the set $h \subset S$ such that $n \in h$ and for all $s_1, s_2 \in h$ either $s_1 = s_2$, $s_1 \prec s_2$ or $s_2 \prec s_1$.

Definition 8
A *choice state* between distinct histories $h_1, h_2 \subset \mathcal{F}$ is the maximal state in the intersection of h_1 and h_2.

We can notice that the minimal possible choice state for two distinct histories is the special state n.
The Definition 6 forces every model of DBL to have its states connected in some way to the present. They are either the consequences of the present state or the causes for it. This is related to our earlier claim that all the future and past states are just representations of information contained in the present state.

Definition 9
The model $\mathfrak{M}$ is the pair $\langle \mathcal{F}, v \rangle$ with the frame $\mathcal{F}$ and the valuation of atomic formulas v.

Similarly as in other dynamic logics, we will change the model in course of the evaluation of formulas. The three possible actions present three possible ways how to do so.

The construction C creates from the model $\mathfrak{M}$ a temporally successive model. Reconstruction R, on the other hand, presents a model that is temporally preceding our current model $\mathfrak{M}$. Both steps make use of the temporal operators in the current model.

Definition 10

We define the model $\mathfrak{M} \mid \mathrm{C}$ for a model $\mathfrak{M}$, as the model where we choose one of the $s \in S$, s.t. $n \prec s$, as the new now, i.e. $n_{\mathrm{C}} = s$. The new model contains only histories that contain n_{C}.

Definition 11

We define the model $\mathfrak{M} \mid \mathrm{R}$ for a model $\mathfrak{M}$, as the model where we choose one of the $s \in S$, s.t. $s \prec n$, as the new now, i.e. $n_{\mathrm{R}} = s$. The new model contains all the histories that contain n_{C}.

Definition 12

We define the model $\mathfrak{M} \mid \varphi$ for a model $\mathfrak{M}$, state s, as the model where for every h it holds that $\mathfrak{M}, s, h \Vdash \mathit{Sett} : \varphi$.

Definition 13 Model satisfies formula - DBL

For the model $\mathfrak{M} = \langle S, \leq, v \rangle$. Where v is the valuation v : Atoms $\rightarrow \mathcal{P}(S)$. For a given state s and history h, st. $s \in h$:

$\mathfrak{M}, s, h \Vdash p$		iff $s \in v(p)$
$\mathfrak{M}, s, h \Vdash \neg\varphi$		iff not $\mathfrak{M}, s, h \Vdash \varphi$
$\mathfrak{M}, s, h \Vdash \varphi \wedge \psi$		iff $\mathfrak{M}, s, h \Vdash \varphi$ and $\mathfrak{M}, s, h \Vdash \psi$
$\mathfrak{M}, s, h \Vdash F\varphi$		iff there is $s' \in S$ s.t. $s \prec s'$ and $\mathfrak{M}, s, h \Vdash \varphi$
$\mathfrak{M}, s, h \Vdash P\varphi$		iff there is an $s' \in S$ s.t. $s' \prec s$ and $\mathfrak{M}, s, h \Vdash \varphi$
$\mathfrak{M}, s, h \Vdash G\varphi$		iff for all $s' \in S$ s.t. $s \prec s'$ it holds $\mathfrak{M}, s, h \Vdash \varphi$
$\mathfrak{M}, s, h \Vdash H\varphi$		iff for all $s' \in S$ s.t. $s' \prec s$ it holds $\mathfrak{M}, s, h \Vdash \varphi$
$\mathfrak{M}, s, h \Vdash \mathit{Sett} : \varphi$		iff there is $s' \in S$ such that $s \prec s'$, for all h' that contain s' it holds $\mathfrak{M}, s', h' \Vdash \varphi$
$\mathfrak{M}, s, h \Vdash [\mathfrak{C}]\varphi$		iff there is a model $\mathfrak{M} \mid \mathrm{C}$ such that $\mathfrak{M} \mid \mathfrak{C}, s, h \Vdash \varphi$
$\mathfrak{M}, s, h \Vdash [\mathfrak{R}]\varphi$		iff for the model $\mathfrak{M} \mid \mathrm{R}$ it holds that $\mathfrak{M} \mid \mathfrak{R}, s, h \Vdash \varphi$
$\mathfrak{M}, s, h \Vdash [\psi]\varphi$		iff there is a model $\mathfrak{M} \mid \psi$ such that $\mathfrak{M} \mid \psi, s, h \Vdash \varphi$

We see therefore that semantics for DBL can work in a very similar fashion as those of BST, because it maintains the same branching structure. However, it adds the element of changing a model with actions. We can close our account with two straightforward theorems[6].

Theorem 14

There exists a BST structure that is not a DBL structure.

Proof. In order for a BST structure to be a DBL structure, we need a to fulfill the definition 6. For every BST structure it holds that $\leq$ is transitive because it is a partial ordering on OW. Nevertheless, the second condition does not hold necessarily. Let us have a BST structure with at least two $e_1, e_2 \in OW$ such that $e_1 \not\leq e_2$ $e_2 \not\leq e_1$. Then if

[6] The suggestion to enhance the paper with at least some of these theorems comes from the reviewer.

either one of these two point-events is n the second condition from the definition does not hold. Thus we have a structure of BST that is not a DBL structure.

We can also construct such a BST structure that for every e there will be a e' such that they are incomparable by $\leq$. Then at least for one e it will be true that $e = n$ and there will be an e' such that they are incomparable, i.e. it is not a DBL structure. □

This answer is not a big surprise if we realize that simultaneous point-events are a valid possibility on BST models. However, they are a crucial impossibility for DBL.

Theorem 15

There exists a DBL structure that is not a BST structure.

Proof. The relation $\prec$ does not have to be dense and this would contradict the first requirement of BST models. An example of a DBL model that has a discrete $\prec$ relation is the Platonia model presented by Barbour. □

5. Discussion

The comparison of DBL and BST is quite straight forward. We see that DBL attempts to capture the presentist idea by creating new models every time the state of the world changes in time (either progresses or regresses). BST on the other hand has a stable unchanging structure that was fixed at the moment of the first setting up of the model. Both models represent branches and hence an option for indeterminism, there can be states in DBL with multiple future options.

There is no big difference in the approach to branching. Already the original BST claimed clearly that there is no backward branching because it is 'plausible enough to warrant making clear what it comes to' (Belnap, 2003). DBL could in theory have backward branching. Nevertheless, for the same reason as BST, the models were chosen not to contain backward branches. For any event that would lead to two possible futures would have a unique reconstruction and therefore it seems plausible to assume there is only one past. Branching as it was familiar from BST is also present in DBL, however, it is contained in single models and does not represent a single model.

One might ask, why did we start out with BST and basically strip it of all its new features, just to end up with a strange branching time model. The answer is that we tried to keep in mind the fruits that came with BST and attempted to maintain them also in our model. This is visible on the fact, how DBL models are based on observers and do not represent some general branching of time.

6. Summary

We showed how Branching space-times approach the question of time and why they do so as an eternalist theory. We then presented a dynamic branching logic that presents a presentist alternative. The comparison of the two approaches showed that they should be equivalent in their expressive strength.

Further investigation could show whether the closeness of DBL to other modal logics gives it an advantage and allows it to prove (as opposed to BST) completeness and similar properties. Especially a deeper investigation should be done in relation to the more philosophical approach of Belnap (Belnap et al., 2001) and Müller (Müller, 2005).

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